Name: ____________________ Date: __________

BASIC TRIGONOMETRIC EQUATION SOLVING
ALGEBRA 2 WITH TRIGONOMETRY

Most of this unit will be devoted to solving equations that contain either the sine or cosine function (and sometimes both!). This process will be relatively straightforward as long as the relationship between an angle and its reference is understood.

***Exercise* #1:** In each of the following, an angle is given. Determine its reference angle and then use your calculator to find the sine and cosine of the angle and its reference. Round your sine and cosine values to the nearest *hundredth* when applicable.

(a) $\theta = 120^\circ$ (b) $\theta = 200^\circ$ (c) $\theta = 310^\circ$

As we can see in *Exercise* #1, the trigonometric functions for any angle have the same **absolute value** as those for its reference. We can exploit this fact to solve basic trigonometric equations.

***Exercise* #2:** Consider the trigonometric equation $\sin(\theta) = -\frac{1}{3}$ over the interval $0^\circ \le \theta \le 360^\circ$.

(a) Determine the reference angle for this problem by evaluating $\sin^{-1}\left(\frac{1}{3}\right)$. Round to the nearest *tenth* of a degree.

(b) In what quadrants is the sine function negative?

(c) Draw rotation diagrams for the two angles that terminate in the quadrants specified in (b) that have the reference angle from (a).

(d) Using your answers from (a), (b) and (c), solve the equation $\sin(\theta) = -\frac{1}{3}$ over the interval $0^\circ \le \theta \le 360^\circ$. State your answers to the nearest *tenth* of a degree.

SOLVING BASIC TRIGONOMETRIC EQUATIONS – FOUR STEPS

1. Determine the reference angle using the inverse sine or inverse cosine.
2. Determine the quadrants in which the terminal rays of the solution lie.
3. Draw a rotation diagram for each solution (if needed).
4. Calculate the solutions to the equation based on #1 to 3.

Exercise #3: Solve each of the following trigonometric equations on the interval $0^\circ \le \theta \le 360^\circ$. Round all answers to the nearest *tenth* of a degree where applicable.

(a) $\cos\theta = \frac{2}{3}$

(b) $\sin\theta = \frac{1}{2}$

(c) $\cos\theta = -0.47$

Any linear trigonometric equation, that is one where the sine and cosine are only raised to the first power, can now be solved by rearranging it into these simple forms.

Exercise #4: Solve each of the following trigonometric equations on the interval $0 \le \alpha \le 360^\circ$. Round your answers to the nearest *tenth* of a degree where applicable.

(a) $5\sin\alpha - 2 = 0$

(b) $2\cos\alpha + 1 = 0$

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BASIC TRIGONOMETRIC EQUATION SOLVING
ALGEBRA 2 WITH TRIGONOMETRY - HOMEWORK

SKILLS

1. Which of the following represents the solution set to $\sin(x) = \frac{1}{2}$ on the interval $0^\circ \le x \le 360^\circ$?

(1) $\{30^\circ, 60^\circ\}$ (3) $\{60^\circ, 120^\circ\}$

(2) $\{30^\circ, 150^\circ\}$ (4) $\{45^\circ, 135^\circ\}$

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2. For the interval $0^\circ \le \theta \le 360^\circ$, which of the following is *not* a solution to $\sin(\theta) = 0$?

(1) $\theta = 360^\circ$ (3) $\theta = 0^\circ$

(2) $\theta = 180^\circ$ (4) $\theta = 90^\circ$?

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3. Accurate to the nearest degree, the solution set of $\cos\alpha = -\frac{3}{5}$ on the interval $0 \le \alpha \le 360^\circ$ is

(1) $\{127^\circ\}$ (3) $\{127^\circ, 233^\circ\}$

(2) $\{53^\circ, 127^\circ\}$ (4) $\{53^\circ, 307^\circ\}$

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4. Expressed as a function of a positive acute angle, $\sin(220^\circ)$ is equivalent to

(1) $\sin(40^\circ)$ (3) $-\sin(60^\circ)$

(2) $-\sin(40^\circ)$ (4) $-\sin(140^\circ)$

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5. Which of the following constitutes the solution set to $2\sin x + \sqrt{2} = 0$ on the interval $0 \le x \le 360^\circ$?

(1) $\{225^\circ, 315^\circ\}$ (3) $\{-45^\circ, 225^\circ\}$

(2) $\{210^\circ, 330^\circ\}$ (4) $\{130^\circ, 230^\circ\}$

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6. Which of the following is a solution to the equation $\sec(\phi) = 2$?

(1) $\phi = 45^\circ$ (3) $\phi = 120^\circ$

(2) $\phi = 90^\circ$ (4) $\phi = 60^\circ$

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7. Solve each of the following trigonometric equations for all values of θ on the interval $0^\circ \le \theta \le 360^\circ$. Express any non-integer answers to the nearest *tenth* of a degree.

(a) $6\sin\theta + 3 = 0$

(b) $10\cos\theta - 1 = 0$

(c) $2\cos\theta + \sqrt{2} = 0$

(d) $3\sin\theta - 1 = 0$

REASONING

8. Explain why the trigonometric equation $2\sin\theta - 5 = 0$ fails to have solutions.

9. Consider the factored trigonometric equation $(\sin\theta - 1)(2\sin\theta + 1) = 0$.

(a) Based on the Zero Product Law, write two linear trigonometric equations whose solutions would comprise the solution set of the original equation.

(b) Find the solution set to the original equation on the interval $0^\circ \le \theta \le 360^\circ$.